Kinetic Theory of Gases

Items Included:

1. "Kinetic Theory of Gases" Model
2. Metal Spheres
3. Cardboard discs – 4
4. Styrofoam Pistons – 2
5. Instructions

Other Items Required:

1. DC Power supply 6-12 Volts
2. Connecting Leads with 4mm plugs at both ends
3. Marking Pencil
4. Small Styrofoam balls

Theory:

The conversion of heat energy into work implies that heat is a form of energy and is intimately related to the motion of particles (or molecules) of which the matter is composed. In case of gases, its macroscopic properties (such as pressure, temperature etc.) are explained in terms of behavior of the molecules by kinetic theory of gases (or Kinetic Molecular Model of gases), using the concept of ideal gas.

According to kinetic theory, every substance, whether it is a solid, liquid or gas, consists of very large number of discrete particles called molecules. Thus, molecule is the smallest entity (or particle) of a substance, which is capable of existing in free state and possesses all the characteristic properties of the substance. The molecules of a substance are in a state of continuous motion and this motion of molecules depends on the temperature of the substance. In case of solids, the molecules are very closely packed due to large forces of cohesion among them. As a result, they are unable to move freely, but can only vibrate about their mean position. This explains the definite shape and volume of solids. The molecules of a liquid are comparatively further apart from each other as compared to solids with very small forces of cohesion among them. Thus, they can move anywhere within the liquid but cannot leave the liquid. Consequently, liquids have definite volume but no definite shape. They acquire the shape of the container they are poured into. In gases, the molecules are much farther apart with negligible forces of attraction among them. They are free to move about in the entire space available to them. Therefore, gases neither have definite shape nor volume.

The study of kinetic theory was first approached by applying it to the matter in the gaseous state, which is governed by simple and definite laws. For the first time,

Bernoulli, in 1730, was able to explain Boyle's law by molecular motions, and for this contribution, he is considered as the father of the kinetic theory. This was followed by substantial contribution in the development of kinetic theory of gases by Clausius, Maxwell, Boltzman, Jeans, Van der Waals, Lorentz and Rayleigh, which covered almost the entire field of the different physical phenomena related to gases. Attempts have been made to extend the application of kinetic theory to liquids and gases, but with little success. Hence, kinetic theory of liquids and solids is much more complicated and still remains in a comparatively undeveloped state.

The discussion of kinetic theory of gases is based on some assumptions, first stated by Clausius.

(a) A closed vessel of a known volume 'V' contains very large number 'N' of molecules, which are in the state of constant motion within the vessel.

(b) These molecules of gas are considered to be rigid, perfectly elastic, solid spheres, identical in all respects such as mass 'm', form etc. These molecules behave as point particles and are of negligible size as compared to the average distance between them and to the dimensions of the vessel containing them.

(c) The molecules are in a state of incessant random motion in all directions with all possible velocities. The gas is said to be in a state of molecular chaos. The molecules obey Newton's laws of motion. Each molecule collides occasionally with the wall of the vessel and these collisions are perfectly elastic (i.e., obey laws of conservation of momentum as well as energy).

(d) With each collision of a molecule with another molecules or walls of the vessel, its velocity is altered in direction and magnitude. Yet, in the steady state, these collisions do not affect the molecular density of gas, i.e., number of molecules per unit volume of gas remains constant.

(e) The collisions are perfectly elastic and there are no forces of attraction or repulsion between the molecules. This implies that the energy contained in the gas molecules is all kinetic.

(f) The time spent in a collision is negligible as compared to the time during which, molecules are moving independently.

(g) Between successive collisions, the molecules move in straight line with uniform velocity. The distance between two collisions is called free path of the molecule and average distance traveled by a molecule between successive collisions is known as the mean free path.

(h) The walls of the vessel are perfectly rigid and infinitely massive and do not move.

Based on the above assumptions, a general theory depicting the behavior common to all gases can be formed. Hence, it should not include any factors that may vary from one gas to another. With these postulates, bulk properties of gases can be explained by the application of laws of probability to a system in dynamic equilibrium, i.e., on the basis of most probable behavior of the largest number of molecules in a given volume, as explained below.

Pressure of a Gas: As discussed above, the molecules of a gas enclosed in a vessel are in random motion, and collide not only with one another but also impinge on the walls of the containing vessel. Each collision of the molecule with the walls of the vessel results in the transfer of a definite momentum to the walls. Since the number of molecules colliding with the walls is very large, the walls will experience almost continuous force. This force acting in a unit area of wall is referred to as pressure of the gas. Thus, according to the kinetic theory of gases, the pressure exerted by a gas is simply the total momentum transmitted to the walls of the containing vessel per second per unit area due to the impact of gas molecules on the walls.

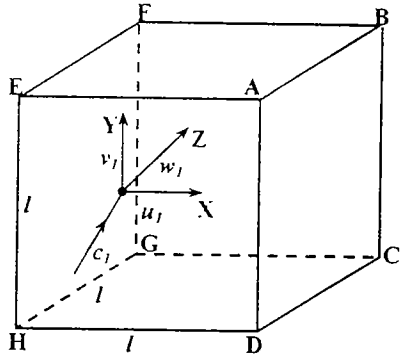


Diagram 1

Let the gas be contained in a cube of side 'l' and 'N' be the number of molecules of gas. Consider a single molecule having mass 'm' and velocity 'c_1' along a certain direction. This velocity can be resolved in three orthogonal components u_1, v_1 and w_1 along three co-ordinate axis X, Y and Z (assumed to be parallel to the sides of cube) respectively. Thus,

$$c_1^2 = u_1^2 + v_1^2 + w_1^2 \qquad \ldots\ldots\ldots (1)$$

If this molecule strikes wall ABCD of the vessel, it will communicate momentum mu_1 to this face. Since, the collision is perfectly elastic, the molecule will rebound

back from the wall with the same velocity u_1. The change in momentum of molecule produced by the impact becomes

$$mu_1 - (-mu_1) = 2mu_1$$

After recoil, this molecule travels to the opposite wall EFGH, similarly collides with it and travels back to the face ABCD again, after covering a distance '$2l$'.

Therefore, the time between two successive collisions on face ABCD is $\frac{2l}{u_1}$.

Thus, the number of collisions per second that the molecule makes with the wall ABCD = $\frac{u_1}{2l}$.

Hence, rate of change of momentum = $2mu_1 \times \frac{u_1}{2l} = \frac{mu_1^2}{l}$

According to Newton's second law of motion, *rate of change of momentum = force applied*. If 'f_1' is the force applied, then

$$f_1 = \frac{mu_1^2}{l}$$

Similarly, the force on wall ABCD 'F_x' due to impact of 'N' molecules is given by

$$F_x = \frac{m}{l}\left(u_1^2 + u_2^2 + \ldots\ldots\ldots + u_N^2\right)$$

Pressure exerted on wall ABCD due to this force is

$$P_x = \frac{F_x}{l^2} = \frac{m}{l^3}\left(u_1^2 + u_2^2 + \ldots\ldots\ldots + u_N^2\right)$$

Similarly, pressure 'P_y' and 'P_z' exerted by the gas on other faces AEFB and BCHF of vessel, which are perpendicular to Y and Z axes respectively, is given as

$$P_y = \frac{m}{l^3}\left(v_1^2 + v_2^2 + \ldots\ldots\ldots + v_N^2\right)$$

$$P_z = \frac{m}{l^3}\left(w_1^2 + w_2^2 + \ldots\ldots\ldots + w_N^2\right)$$

Since, pressure exerted by the gas is same in all directions, the average pressure 'P' of the gas is given by

$$P = \frac{P_x + P_y + P_z}{3}$$

$$= \frac{m}{3l^3}\left(u_1^2 + u_2^2 + \ldots\ldots\ldots + u_N^2\right) + \left(v_1^2 + v_2^2 + \ldots\ldots\ldots + v_N^2\right) + \left(w_1^2 + w_2^2 + \ldots\ldots\ldots + w_N^2\right)$$

Also, Volume of the cube = $V = l^3$; and from equation (1), $c_1^2 = u_1^2 + v_1^2 + w_1^2$, $c_2^2 = u_2^2 + v_2^2 + w_2^2$ and so on.

$$\Rightarrow P = \frac{1}{3}\frac{m}{V}\left(c_1^2 + c_2^2 + \ldots\ldots\ldots + c_N^2\right) = \frac{1}{3}\frac{mN}{V}C^2 \qquad \ldots\ldots\ldots (2)$$

Where, $C^2 = \frac{c_1^2 + c_2^2 + \ldots\ldots\ldots + c_N^2}{N}$, and is known as the *mean square velocity* of the molecules.

If 'M' be the total mass of the gas, then $M = mN$. Thus, the equation (2) becomes,

$$P = \frac{1}{3}\frac{M}{V}C^2 \qquad \ldots\ldots\ldots (3)$$

Root Mean Square (RMS) Velocity: 'C' as discussed above, is called root mean square velocity of the molecules and is equal to the square root of mean square of the velocities of the individual molecules. 'C' is not the same as mean of the velocities of different molecules. It can be computed from equation (3) by knowing pressure 'P' and density 'ρ' of the gas.

Density of the gas = $\rho = \frac{M}{V}$

$$\therefore P = \frac{1}{3}\rho C^2 \qquad \ldots\ldots\ldots (4)$$

$$\Rightarrow C = \sqrt{\frac{3P}{\rho}} \qquad \ldots\ldots\ldots (5)$$

If 'E' is the Mean kinetic energy per unit volume of the gas, then E = $\frac{1}{2}\rho C^2$.

Thus, from equation (4)

$$P = \frac{2}{3} \times \frac{1}{2}\rho C^2 = \frac{2}{3}E \qquad \ldots\ldots\ldots (6)$$

Hence, pressure of a gas is numerically equal to two third of the mean kinetic energy per unit volume of the gas molecules.

Kinetic Interpretation of Temperature: It was proved by Avogadro that equal volumes of all gases contain equal number of molecules at the same temperature. According to him, one mole of all gases weigh equal to their molecular masses expressed in grams (or gram molecular weight). Also, one mole of different gases contained in vessels of identical volume at same temperature exert equal pressure on the walls of the vessels.

If 'V' is the volume occupied by gram molecule of the gas and 'M' be the molecular weight, equation (3) can also be expressed as

$$PV = \frac{1}{3}MC^2 \qquad \ldots\ldots\ldots (7)$$

According to ideal gas equation,

$$PV = nRT \qquad \ldots\ldots\ldots (8)$$

Where, 'n' is the number of moles of gas under observation contained in a vessel. 'R' is the universal gas constant having value 8.31 Joules mol-K, and 'T' is the absolute temperature of gas in Kelvin. Here, since we have taken 1 mole of gas, n=1. Thus, from equations (7) and (8), we get

$$\frac{1}{3}MC^2 = RT$$

$$\therefore \Rightarrow C^2 = \frac{3R}{M} \times T \qquad \ldots\ldots\ldots (9)$$

Let 'N' be the number of molecules contained in one mole of gas (or Avogadro's constant having value 6.0225×10^{23}). Dividing numerator and denominator of above equation by 'N', we get

$$C^2 = 3\frac{\frac{R}{N}}{\frac{M}{N}} \times T = \frac{3k}{m}T \qquad \ldots\ldots\ldots (10)$$

Where, 'm' is the mass of a molecule of gas and 'k' is the Boltzmann's gas constant = $\frac{R}{N}$.

Equation (10) shows that the mean square velocity 'C^2' is directly proportional to the absolute temperature 'T'. At T = 0, C^2 = 0, or C = 0. Hence, the absolute zero

of temperature is the temperature at which the molecular translational velocities of a gas are reduced to zero. Also, from equation (10),

$$\frac{1}{2}mC^2 = \frac{3}{2}kT \qquad \ldots\ldots\ldots (11)$$

Above equation shows that the mean kinetic energy per molecule in a given mass of gas is proportional to the absolute temperature of the gas and does not depend on the pressure, volume or kind of molecule. In other words, to every uniform temperature, there is a corresponding uniform distribution of kinetic energy and the molecular collisions provides the way that leads to the equalization of temperature.

Experimental Confirmation of the Kinetic Theory of Gases: The basic fundamental of kinetic theory of gases is the motion of molecules due to thermal agitation. This motion of molecules is so rapid and irregular that the gas on the whole remains motionless. This theory has adequately explained various thermal and allied phenomena, such as Boyle's law, Charle's law, Ideal gas law, Avogadro's law, Gay Lussac's law, Regnault's law, Dalton's law of partial pressure, Graham's law of diffusion etc. But, these explanations are more or less indirect evidence of the validity of the theory, since various assumptions of the kinetic theory have been introduced on empirically based results. There are two direct and independent experimental evidences that are employed to support the validity of this theory, viz., Brownian movement of gas molecules and Width of spectral lines and visibility of interference fringes. Brownian movement has been discussed in the following discussion whereas, the width of spectral lines and visibility of interference fringes, being outside the scope of present discussion, has not been discussed.

Brownian Movement: The phenomenon of Brownian movement has been named after the English botanist Robert Brown, who first discovered it in 1827. His observation described the motion of particles to be rapid, incessant and unabated in an irregular fashion in all directions. He observed that the particles in a liquid spin, rises, sinks and rises again, without ever coming to rest, the motion being sudden, random and unpredictable; the smaller the particle, the more lively and noticeable are its movements. Initially, the Brownian movement was attributed to the surface tension, irregularities in the temperature of fluid, vibrations, chemical actions etc. All these explanations proved to be inadequate for complete explanation of this phenomenon. Later, it was realized that this phenomenon is the result of thermal agitation of the molecules of fluid. The irregularity of motion of particles implies similar irregularities in the molecules of fluids. The effect can only be obtained with very small particles due to two reasons. Firstly, the kinetic energy of the molecules is so small that they cannot communicate a finite velocity

to a large particle. The second reason is based on Maxwell's distribution of velocities among the molecules. Maxwell showed that the velocities are distributed among molecules according to a definite law and this distribution is true for any small element of volume in the substance. Hence, if the particle is large, the force due to impact of molecules on one side will be just counterbalanced by that due to those on the other side and net force on the particle will be zero. But, in the case of a smaller particle, it suffers encounter with very few molecules at a given time and the irregularities in the motion of molecules will begin to manifest themselves. Thus, we have a direct evidence of motion of molecules due to thermal agitation, which forms the basic postulate of the kinetic theory.

Deviation of Real Gas Behavior from that of an Ideal Gas: In case of kinetic theory of gases explaining behavior of ideal gases, it has been assumed that

(a) The molecules do not exert any force on one another or on the walls of the vessel except during actual collisions.

(b) The molecules are point particles, whose volume has been neglected as compared to the volume of vessel.

(c) The collisions of molecules with one another and with the walls of container are perfectly elastic.

In fact, no real gas fulfils these conditions. As such, the behavior of real gas shows deviations from that of an ideal gas (as explained on the basis of kinetic theory). These deviations become quite substantial as the temperature of gases approaches their condensation point. The real gases behave like ideal gas with sufficiently high degree of accuracy at very low densities and when their temperatures are far above their condensation temperatures. Under these situations, the above assumptions are not completely valid.

Kinetic Theory of Gases Model Apparatus:

The apparatus consists of clear transparent Perspex cylindrical tube mounted on the circular holder and held firmly with the help of three tightening screws. The circular holder, in turn, is mounted on a heavy metal base with the help of pillars. This circular holder has a rubber diaphragm at its bottom below which is fixed a metal piston. The metal piston is connected to the cam fixed on the motor shaft through a metal strip. The motor is mounted on the metal base. Metal base is also provided with a pair of color-coded 4mm socket terminals for connecting the motor to a low voltage DC power supply. On operating the motor, cam starts rotating. This rotation of cam results in up/down oscillations of the piston through the connecting strip. Oscillations of the piston induce vibrations in the rubber

diaphragm, which in turn, leads to the agitation of spherical metal balls placed on the diaphragm.

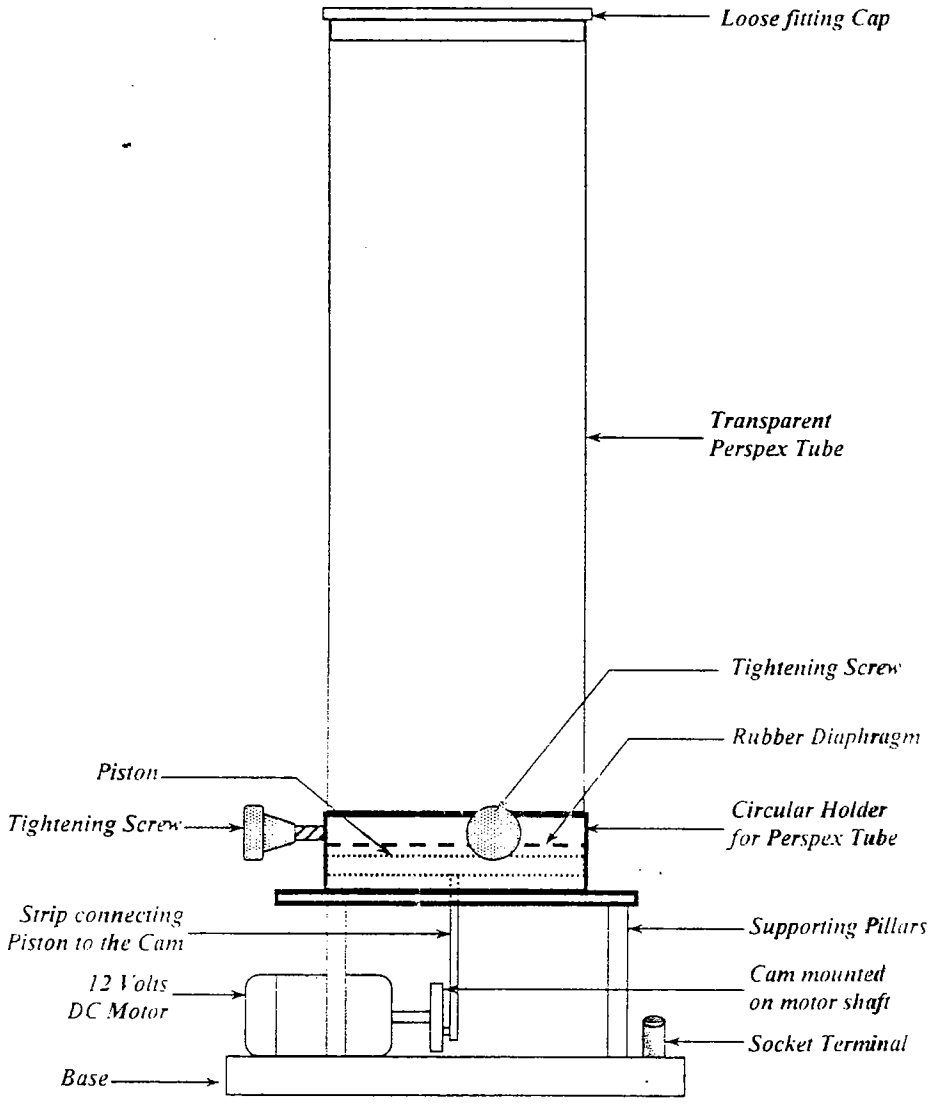


Diagram 2: "Kinetic Theory of Gases" Apparatus

At the top of the cylindrical tube, a loose-fitting cap with perforations is placed. Also included with this apparatus are two Styrofoam pistons, which are used for demonstration of the concept of pressure when they are introduced inside the tube from top. Four cardboard discs are included for loading the Styrofoam piston for the demonstration of Boyle's law. The apparatus requires a 12 volts DC power supply for satisfactory operation. The motion of spherical balls can be increased or decreased by varying the voltage supplied to the motor from the power supply. For good results, it is advised to spread the spherical ball on rubber diaphragm evenly about its center to cover about two-third of the cross-sectional area of diaphragm.

This apparatus is useful for the demonstration of various concepts related to kinetic theory of gases. Some of these concepts are

(a) Motion of molecules in gases and concept of Brownian motion.
(b) Concept of pressure and decrease of atmospheric pressure with the increase in altitude.
(c) Boyle's law.
(d) Distribution of velocities among the molecules.
(e) Concept of thermal kinetic energy of molecules.

Procedure:

1. Place the metal base on level surface.
2. Remove the Perspex tube and place metal spherical balls on the diaphragm so as to cover about two-third of the cross-sectional area of the diaphragm. Mount the tube firmly in the holder using three tightening screws.
3. Place the loose fitting cap on top of the Perspex tube.
4. Connect the red and black terminal provided on the base to the positive and negative of the low voltage DC power supply using connecting leads.
5. Switch on the power supply and set the output to about 12 volts.
6. Rotation of the motor will set the piston in an up down oscillatory motion, which in turn, will produce vibrations in the rubber diaphragm. The vibration of the rubber diaphragm is transferred to the metal spheres placed on it (transmission of kinetic energy from piston to diaphragm and to metal spheres) and agitates them. This leads to the violent (random) motion of metal spheres. Metal spheres can be observed to be dancing up and down haphazardly. The motion of metal spheres, although being random, is not exactly like the motion of molecules of a gas. These metal spheres are under the continuous effect of gravity and the their collisions with one another or with the wall of Perspex tube are not perfectly elastic (their

kinetic energy is not conserved). But, for all practical purposes, the characteristics of their motion are very much similar to that of molecules of a gas.

The loss of kinetic energy of the metal spheres due to collisions is compensated when they fall back on the vibrating rubber diaphragm. The mechanical energy contained in the diaphragm is transferred to the metal spheres as and when they fall on the diaphragm. Thus, motion of metal spheres is such that the loss of kinetic energy suffered by them is continuously being replenished by the vibrating diaphragm. If this is not the case, either the motion of spheres will die down because of continuous loss of energy (if energy supplied by the diaphragm is less) or the motion will keep growing steadily (if energy supplied by the diaphragm is more). As such, the moving metal spheres can be treated as having uniform kinetic energy (analogous to the gas molecules at constant temperature).

7. It can be observed that as we move up from the rubber diaphragm, the number of metal spheres that can reach that height goes on decreasing. There is a particular height where the number of metal spheres that can reach is maximum. This corresponds to the mean energy of the metal spheres. The reducing density of metal spheres as we go higher from the diaphragm demonstrates the thinning of the atmosphere with increase in altitude. The situation is similar to that of the air molecules on earth that are also under the influence of gravity.

8. Stop the apparatus by switching off the power supply. Remove the top metal cap and introduce a Styrofoam piston. It will drop to the bottom of the Perspex tube. As soon as the apparatus is switched on, the Styrofoam piston starts rising under the effect of random motion of metal spheres. The Styrofoam piston rises to a point, where its weight is just balanced by the upward force due to the motion of metal spheres. The motion of these spheres simulates pressure (equal to the force exerted by motion of metal spheres per unit area) that is analogous to the pressure of a gas. On decreasing the voltage output of power supply, the molecular motion will decrease, thus decreasing the pressure exerted by the metal spheres on Styrofoam piston. As a result, the Styrofoam piston will fall below the previous position. Similarly, on increasing the voltage output of power supply, more will be the motion of molecules and Styrofoam piston will rise beyond its previous position. It is evident from the discussion that more violent or agitated is the motion (i.e., more is the energy) of metal spheres (or molecules of a gas), more is the pressure exerted by them on the walls bounding them.

9. For demonstrating Boyle's law, reduce the number of metal spheres inside Perspex tube by half, and start the functioning of apparatus as in previous step after introducing the Styrofoam piston. Mark the height to which piston rises with a marking pencil. This position corresponds to the volume at the pressure exerted by metal spheres. Again repeat this step after increasing the number of metal spheres to double the previous quantity. On running the apparatus again at the same voltage output of power supply, the Styrofoam piston will rise beyond this marked position. Mark the second position of the piston. The second position shows the increased volume. Introduce the second Styrofoam piston so as to load the first Styrofoam piston. The position of the first Styrofoam piston will lower. Increase the loading using cardboard discs, if required, to bring the first piston back to the first marked position. Increasing the number of spheres and loading of Styrofoam piston from top to lower the Styrofoam piston to its initial position indicates the increase in pressure with reduction in volume. This implies that with the increase in pressure, the volume of a gas decreases keeping other factors such as quantity and temperature of gas constant, which is same as that of Boyle's law.

10. For studying the concept of Brownian motion, remove the Styrofoam pistons and introduce Styrofoam balls in the tube. On running the apparatus, the metal spheres (or molecules) can be seen colliding with the Styrofoam balls. This results in the random and bombarding motion of the Styrofoam balls. Although, this motion is not truly accurate enough to depict the Brownian motion due to the combined effects of buoyancy of so-called gas inside the tube consisting of metal spheres and gravity, still it nearly demonstrates large-scale analogue of Brownian motion and gives fair indication of general concepts involved.

Precautions:

1. Apparatus should be placed on a level surface before operation. Uneven or inclined surface may result in improper functioning of apparatus.
2. The apparatus should always be operated with the metal cap placed at its position on top of the Perspex tube.
3. The Perspex tube should not be loosened or removed from its holder when the apparatus is operating.
4. The apparatus should not be operated for prolonged period at voltages much above or below 12 volts to avoid any damage to the motor.