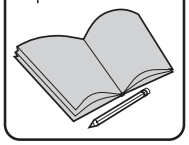
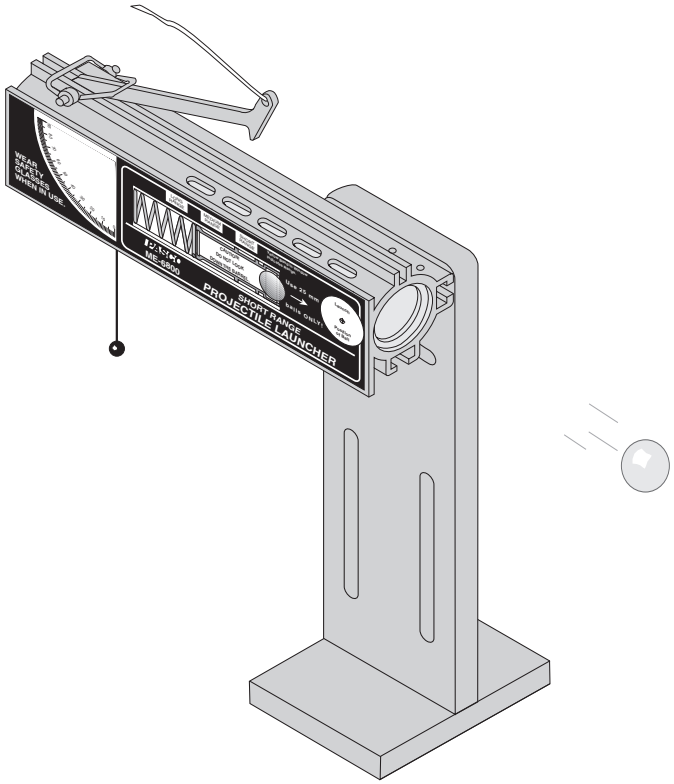
Includes
Teacher's Notes
and
Typical
Experiment Results



Instruction Manual and Experiment Guide for the PASCO scientific Model ME-6800, 6801

012-05043F

Projectile Launcher Short / Long Version



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Table of Contents

Section	Page
Copyright, Warranty, and Equipment Return	ii
Introduction	1
Equipment	2
General Operation Of The Projectile Launcher	3
Installation Of The Optional Photogate Bracket	4
Installing the 2-Dimensional Collision Attachment	5
Expectations For The Projectile Launcher	5
Experiments	
1. Projectile Motion	7
2. Projectile Motion Using Photogates	11
3. Projectile Range versus Angle	15
4. Projectile Path	19
5. Conservation of Energy	23
6. Conservation of Momentum in Two Dimensions	27
7. Varying Angle to Maximize Height on a Wall	31
8. Demo: Do 30· and 60· Give Same Range?	33
9. Demo: Simultaneously Shoot Two Balls Horizontally at Different Speeds	35
10. Demo: Shoot through Hoops	37
11. Teacher's Guide	39

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This manual authored by: Ann Hanks

This manual edited by: Jon Hanks

Teacher's guide written by: Eric Ayars

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Type of Computer (Make, Model, Speed).

Type of external Cables/Peripherals.

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A detailed description of the problem/sequence of events. (In case you can't call PASCO right away, you won't lose valuable data.)

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Part number and Revision (listed by month and year on the front cover).

Have the manual at hand to discuss your questions.

Introduction

The PASCO Projectile Launcher has been designed for projectile experiments and demonstrations. The only additional equipment required is a C-clamp for clamping the Launcher to a table. The features of the Projectile Launcher include:

- LAUNCH AT ANY ANGLE:

  Balls can be launched at any angle from zero to 90 degrees measured from the horizontal. The angle is easily adjusted using thumb screws. The built-in protractor and plumb-bob on the side of the launcher give a convenient and accurate way of determining the angle of inclination.

- THREE RANGE SETTINGS:

  There are three ranges from which to choose. For the Short Range Projectile Launcher these three ranges are approximately 1.2 meters, 3 meters, and 5 meters, when the angle is 45 degrees. For the Long Range Demonstration Projectile Launcher, the three ranges are approximately 2.5 meters, 5 meters, and 9 meters. The difference between these two versions of the Projectile Launcher is the strength of the spring. The Long Range version is intended for large classroom demonstrations.

- FIXED ELEVATION INDEPENDENT OF ANGLE:

  The Projectile Launcher pivots at the muzzle end so the elevation of the ball as it leaves the barrel does not change as the angle is varied. The base has two sets of slots: The top curved slot is used when it is desired to change the angle and the bottom two slots are used when it is desired to shoot horizontally only, such as into a pendulum or a Dynamics Cart.

- REPEATABLE RESULTS:

  There is no spin on the ball since the piston keeps the ball from rubbing on the walls as it travels up the barrel. The sturdy base can be secured to a table with a C- clamp (not included) so there is very little recoil. The trigger is pulled with a string to minimize the jerking.

- BARREL SIGHTS AND SAFETY PRECAUTIONS:

  There are sights for aiming the Projectile Launcher. These sights can be viewed from the back of the Projectile Launcher by looking through the back end of the barrel.

➤ **WARNING:** Never look down the front of the barrel because it may be loaded. To check to see if the ball is in the barrel and whether the Projectile Launcher is cocked, look at the slots in the side of the barrel. Safety goggles are provided. The yellow indicator seen through the side slot indicates the position of the piston and the ball can also be seen through these slots when it is in the piston.

- COMPUTER COMPATIBLE: Photogates may be attached with the accessory bracket (ME-6821 A) to connect the Projectile Launcher to a computer to measure the muzzle speed. Also a photogate at the muzzle and the Time of Flight accessory (ME-6810) can be used to time the flight of the ball.

- COMPACT STORAGE: The Projectile Launcher stores away in a small space. The ramrod attaches to the Projectile Launcher with Velcro® and the Projectile Launcher can be aligned with the base so it takes up the minimum amount of space on the shelf.

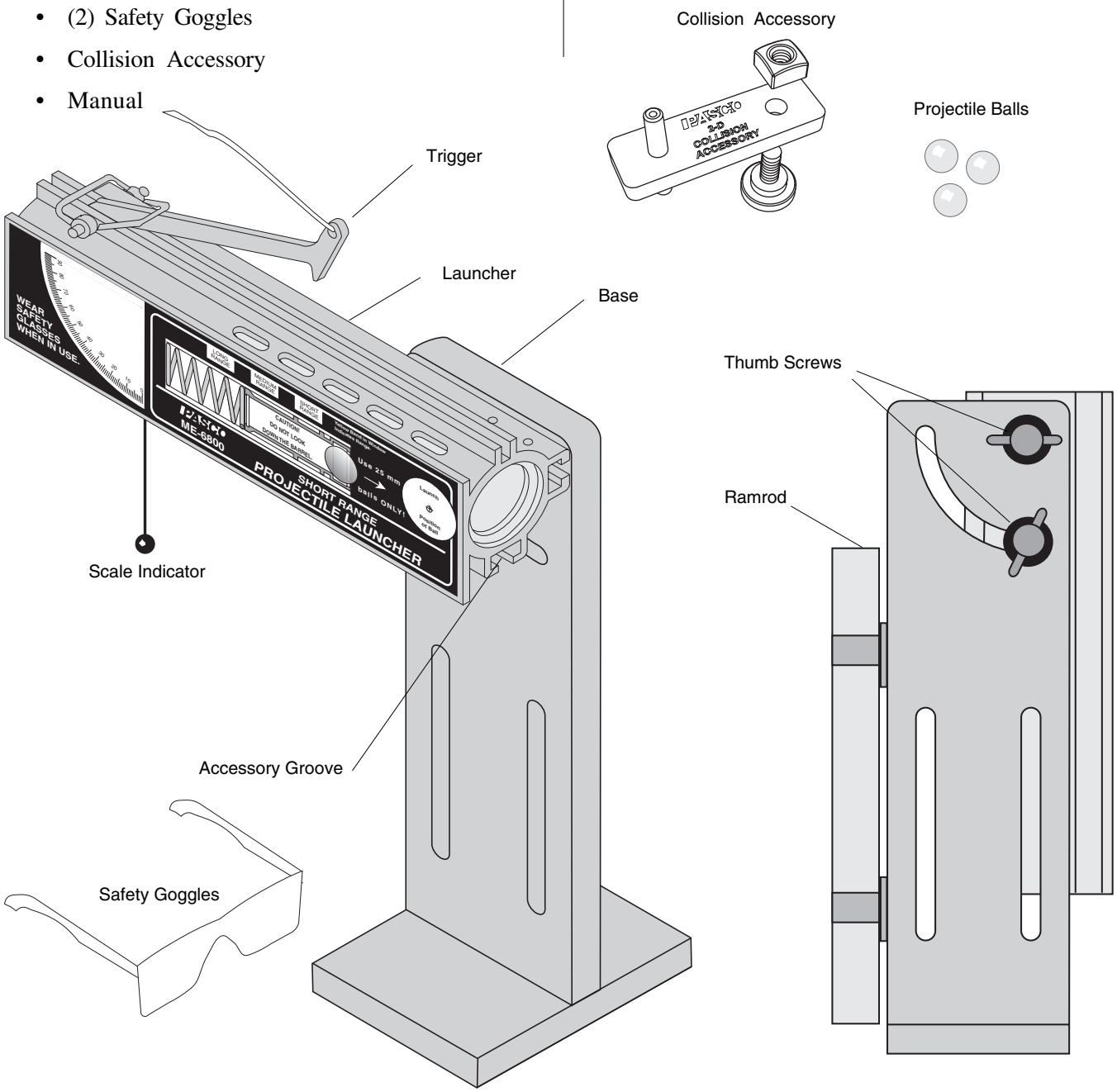
Equipment

The following is a description of the equipment that is included with various models of the Projectile Launcher.

The ME-6800 (Short Range) Projectile Launcher Student/Demo Version includes the following:

- Launcher and Base (Assembled)
- (3) Plastic Balls
- Ramrod (Attached with Velcro® to stand)
- (2) Safety Goggles
- Collision Accessory
- Manual

The ME-6801 (Long Range) Projectile Launcher Student/Demo Version includes the same items as the ME-6800 but is capable of significantly greater projectile range.



General Operation of the Projectile Launcher

① Ready

- Always wear safety goggles when you are in a room where the Projectile Launcher is being used.
- The base of the Projectile Launcher must be clamped to a sturdy table using the clamp of your choice. When clamping to the table, it is desirable to have the label side of the Launcher even with one edge of the table so a plumb bob can be used to locate the position of the muzzle with respect to the floor.
- The Projectile Launcher can be mounted to the bracket using the curved slot when it is desired to change the launch angle. It can also be mounted to the lower two slots in the base if you are only going to shoot horizontally, such as into a pendulum or a Dynamics Cart.

② Aim

- The angle of inclination above the horizontal is adjusted by loosening both thumb screws and rotating the Launcher to the desired angle as indicated by the plumb bob and protractor on the side of the Launcher. When the angle has been selected, both thumb screws are tightened.
- You can bore-sight at a target (such as in the Monkey-Hunter demonstration) by looking through the Launcher from the back end when the Launcher is not loaded. There are two sights inside the barrel. Align the centers of both sights with the target by adjusting the angle and position of the Launcher.

③ Load

- Always cock the piston with the ball in the piston. Damage to the piston may occur if the ramrod is used without the ball.
- Place the ball in the piston. Remove the ramrod from its Velcro® storage place on the base. While viewing the range-setting slots in the side of the Launcher, push the ball down the barrel with the ramrod until the trigger catches the piston at the desired range setting.
- Remove the ramrod and place it back in its storage place on the base.
- When the Projectile Launcher is loaded, the yellow indicator is visible in one of the range slots in the side of the barrel and the ball is visible in another one of the slots in the side of the barrel. To check to see if the Launcher is loaded, always check the side of the barrel. Never look down the barrel!

④ Shoot

- Before shooting the ball, make certain that no person is in the way.
- To shoot the ball, pull straight up on the lanyard (string) that is attached to the trigger. It is only necessary to pull it about a centimeter.
- The spring on the trigger will automatically return the trigger to its initial position when you release it.

⑤ Maintenance and Storage

- No special maintenance of the Projectile Launcher is required.
- Do not oil the Launcher!!
- To store the Launcher in the least amount of space, align the barrel with the base by adjusting the angle to 90 degrees. If the photogate bracket and photogates are attached to the Launcher, the bracket can be slid back along the barrel with the photogates still attached.

Installing the Optional Photogate Bracket (ME-6821)

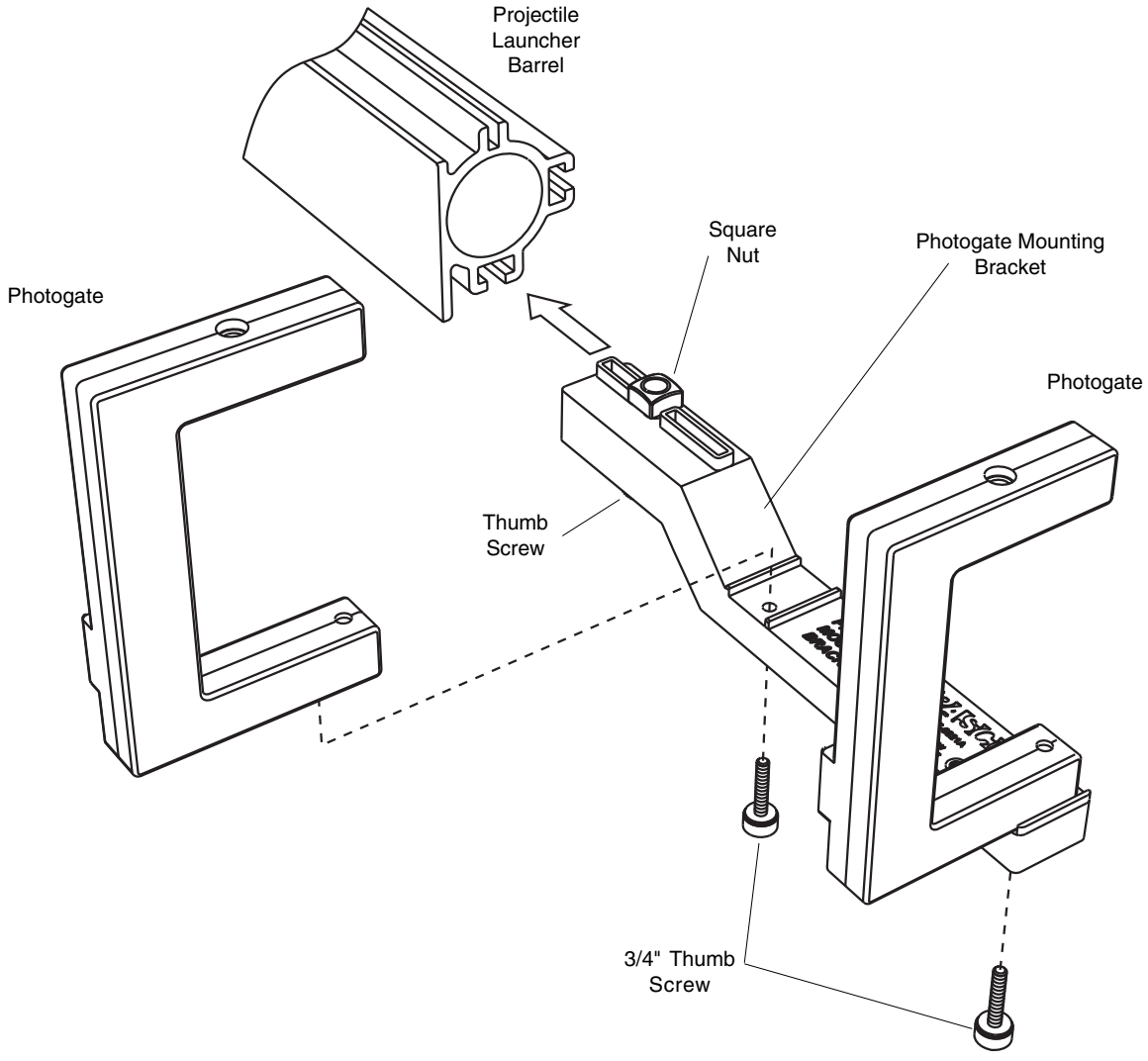
The Photogate Bracket is an optional accessory for mounting one or two photogates on the Projectile Launcher to measure the muzzle velocity of the ball.

Installation is as follows:

① Prepare the bracket by inserting the thumb screw through the hole in the bracket near the end that has the post (see diagram for orientation) and start the square nut onto the end of the thumb screw. Attach the photogates to the bracket using the remaining holes in the bracket and the screws provided with the photogates.

② To mount the bracket to the Launcher, align the square nut in the slot on the bottom of the barrel and slide the nut and the post into the slot. Slide the bracket back until the photogate nearest to the barrel is as close to the barrel as possible without blocking the beam. Tighten the thumb screw to secure the bracket in place.

③ When storing the Projectile Launcher, the photogate bracket need not be removed. It can be slid back along the barrel with or without the photogates in place, making as compact a package as possible.



Installing the 2-Dimensional Collision Accessory

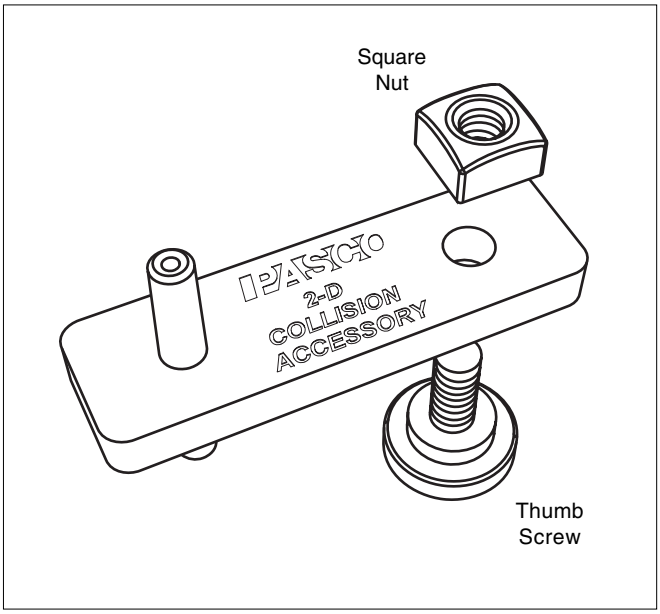
Introduction

The two dimensional Collision Accessory consists of a plastic bar with a thumb screw and square nut. It is used with the Projectile Launcher to hold a second ball in front of the muzzle so the launched ball will collide with the second ball, creating a 2-dimensional collision.

Assembly

To assemble the collision accessory, insert the screw through the hole and secure with the nut as shown below.

To mount the collision ccessory to the Launcher the square nut slides into the T-shaped channel on the bottom of the barrel. (See Figure 6.2 on page 28)



Expectations for the Projectile Launcher

The following are helpful hints and approximate values you may find useful:

① The muzzle speed will vary slightly with angle. The difference between muzzle speed when shot horizontally versus vertically can be anywhere from zero to 8%, depending on the range setting and the particular launcher.

② Although the muzzle end of the Projectile Launcher doesn't change height with angle, it is about 30 cm (12 inches) above table level, so if it is desired to use the simple range formula, it is necessary to shoot to a table that is at the same height as the muzzle.

③ The scatter pattern is minimized when the Projectile Launcher base is securely clamped to a sturdy table. Any wobble in the table will show up in the data.

The angle of inclination can be determined to within one- half of a degree.

Notes

Experiment 1: Projectile Motion

EQUIPMENT NEEDED:

-Projectile Launcher and plastic ball
-Plumb bob
-Meter stick
-Carbon paper
-White paper

Purpose

The purpose of this experiment is to predict and verify the range of a ball launched at an angle. The initial velocity of the ball is determined by shooting it horizontally and measuring the range and the height of the Launcher.

Theory

To predict where a ball will land on the floor when it is shot off a table at some angle above the horizontal, it is necessary to first determine the initial speed (muzzle velocity) of the ball. This can be determined by shooting the ball horizontally off the table and measuring the vertical and horizontal distances through which the ball travels. Then the initial velocity can be used to calculate where the ball will land when the ball is shot at an angle.

HORIZONTAL INITIAL VELOCITY:

For a ball shot horizontally off a table with an initial speed, v_o, the horizontal distance travelled by the ball is given by $x = v_o t$, where t is the time the ball is in the air. Air friction is assumed to be negligible.

The vertical distance the ball drops in time t is given by $y = \frac{1}{2} g t^2$.

The initial velocity of the ball can be determined by measuring x and y. The time of flight of the ball can be found using:

$$t = \sqrt{\frac{2y}{g}}$$

and then the initial velocity can be found using $v_0 = \frac{x}{t}$.

INITIAL VELOCITY AT AN ANGLE:

To predict the range, **x**, of a ball shot off with an initial velocity at an angle, θ, above the horizontal, first predict the time of flight using the equation for the vertical motion:

$$y = y_0 + (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$$

where $\mathbf{y_o}$ is the initial height of the ball and **y** is the position of the ball when it hits the floor. Then use $x = (v_0 \cos\theta)\, t$ to find the range.

Setup

① Clamp the Projectile Launcher to a sturdy table near one end of the table.

② Adjust the angle of the Projectile Launcher to zero degrees so the ball will be shot off horizontally.

Procedure

Part A: Determining the Initial Velocity of the Ball

① Put the plastic ball into the Projectile Launcher and cock it to the long range position. Fire one shot to locate where the ball hits the floor. At this position, tape a piece of white paper to the floor. Place a piece of carbon paper (carbon-side down) on top of this paper and tape it down. When the ball hits the floor, it will leave a mark on the white paper.

② Fire about ten shots.

③ Measure the vertical distance from the bottom of the ball as it leaves the barrel (this position is marked on the side of the barrel) to the floor. Record this distance in Table 1.1.

④ Use a plumb bob to find the point on the floor that is directly beneath the release point on the barrel. Measure the horizontal distance along the floor from the release point to the leading edge of the paper. Record in Table 1.1.

⑤ Measure from the leading edge of the paper to each of the ten dots and record these distances in Table 1.1.

⑥ Find the average of the ten distances and record in Table 1.1.

⑦ Using the vertical distance and the average horizontal distance, calculate the time of flight and the initial velocity of the ball. Record in Table 1.1.

Part B: Predicting the Range of the Ball Shot at an Angle

① Adjust the angle of the Projectile Launcher to an angle between 30 and 60 degrees and record this angle in Table 1.2.

② Using the initial velocity and vertical distance found in the first part of this experiment, assume the ball is shot off at the new angle you have just selected and calculate the new time of flight and the new horizontal distance. Record in Table 1.2.

③ Draw a line across the middle of a white piece of paper and tape the paper on the floor so the line is at the predicted horizontal distance from the Projectile Launcher. Cover the paper with carbon paper.

④ Shoot the ball ten times.

⑤ Measure the ten distances and take the average. Record in Table 1.2.

Analysis

① Calculate the percent difference between the predicted value and the resulting average distance when shot at an angle.

② Estimate the precision of the predicted range. How many of the final 10 shots

Table 1.1 Determining the Initial Velocity

Vertical distance = _____ Horizontal distance to paper edge = _____

Calculated time of flight = _____ Initial velocity = _____

Trial Number	Distance
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Average	
Total Distance	

Table 1.2 Confirming the Predicted Range

Angle above horizontal = _____ Horizontal distance to paper edge = _____

Calculated time of flight = _____ Predicted Range = _____

Trial Number	Distance
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Average	
Total Distance	

Notes

Experiment 2: Projecile Motion Using Photogates

EQUIPMENT NEEDED

-Projectile Launcher and plastic ball
-(2) photogates
-Plumb bob
-Carbon paper
-Photogate bracket
-Computer
-Meter stick
-White paper

Purpose

The purpose of this experiment is to predict and verify the range of a ball launched at an angle. Photogates are used to determine the initial velocity of the ball.

Theory

To predict where a ball will land on the floor when it is shot off a table at some angle above the horizontal, it is necessary to first determine the initial speed (muzzle velocity) of the ball. This can be determined by shooting the ball and measuring the speed using photogates. To predict the range, x, of the ball when it is shot off with an initial velocity at an angle q, above the horizontal, first predict the time of flight using the equation for the vertical motion:

$$y = y_0 + (v_0 \sin\theta)\, t - \frac{1}{2}\, gt^2$$

where y_o is the initial height of the ball and y is the position of the ball when it hits the floor. Then use $x = (v_0 \cos\theta)\, t$ to find the range.

Set-Up

① Clamp the Projectile Launcher to a sturdy table near one end of the table.

② Adjust the angle of the Projectile Launcher to an angle between 30 and 60 degrees.

③ Attach the photogate bracket to the Launcher and attach two photogates to the bracket. Plug the photogates into a computer or other timer.

Procedure

PART A: Determining the Initial Velocity of the Ball

① Put the plastic ball into the Projectile Launcher and cock it to the long range position.

② Run the timing program and set it to measure the time between the ball blocking the two photogates.

③ Shoot the ball three times and take the average of these times. Record in Table 2.1.

Using that the distance between the photogates is 10 cm, calculate the initial speed and record it in Table 2.1.

Table 2.1 Initial Speed

Trial Number	Time
1	
2	
3	
Average Time	
Initial Speed	

landed within this range?

PART B: Predicting the Range of the Ball Shot at an Angle

① Keep the angle of the Projectile Launcher at the chosen angle.

② Measure the vertical distance from the bottom of the ball as it leaves the barrel (this position is marked on the side of the barrel) to the floor. Record this distance in Table 2.2.

③ Using the initial velocity and vertical distance found, assume the ball is shot off at the angle you have selected and calculate the time of flight and the horizontal distance. Record in Table 2.2.

④ Draw a line across the middle of a white piece of paper and tape the paper on the floor so the line is at the predicted horizontal distance from the Projectile Launcher. Cover the paper with carbon paper.

⑤ Shoot the ball ten times.

⑥ Measure the ten distances and take the average. Record in Table 2.2.

Table 2.2 Confirming the Predicted Range

Angle above horizontal = _____

Horizontal distance to paper edge = _____

Calculated time of flight= _____

Predicted Range = _____

Trial Number	Distance
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Average	
Total Distance	

Analysis

① Calculate the percent difference between the predicted value and the resulting average distance when shot at an angle.

② Estimate the precision of the predicted range. How many of the final 10 shots landed within this range?

Notes

Experiment 3: Projectile Range Versus Angle

EQUIPMENT NEEDED

-Projectile Launcher and plastic ball
-measuring tape or meter stick
-box to make elevation same as muzzle
-graph paper
-plumb bob
-carbon paper
-white paper

Purpose

The purpose of this experiment is to find how the range of the ball depends on the angle at which it is launched. The angle that gives the greatest range is determined for two cases: for shooting on level ground and for shooting off a table.

Theory

The range is the horizontal distance, x, between the muzzle of the Launcher and the place where the ball hits, given by $x = (v_0 \cos\theta)t$, where v_0 is the initial speed of the ball as it leaves the muzzle, θ is the angle of inclination above horizontal, and t is the time of flight. See figure 3.1.

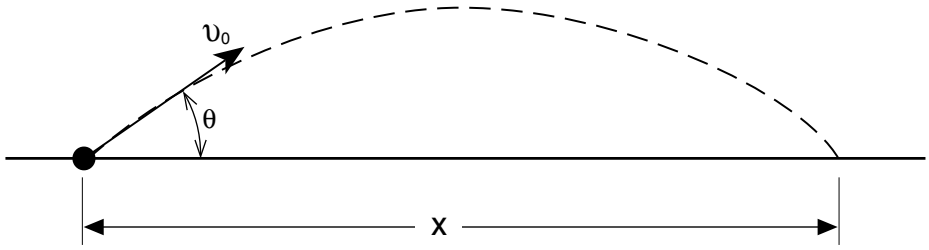


Figure 3.1 Shooting on a level surface

For the case in which the ball hits on a place that is at the same level as the level of the muzzle of the launcher, the time of flight of the ball will be twice the time it takes the ball the reach the peak of its trajectory. At the peak, the vertical velocity is zero so

$$v_y = 0 = v_0 \sin\theta - g t_{peak}$$

Therefore, solving for the time gives that the total time of flight is $t = 2t_{peak} = 2\,\frac{v_0 \sin\theta}{g}$.

For the case in which the ball is shot off at an angle off a table onto the floor (See Figure 3.2) the time of flight is found using the equation for the vertical motion:

$$y = y_0 + (v_0 \sin\theta)\, t - \frac{1}{2}\, g t^2$$

where y_o is the initial height of the ball and y is the position of the ball when it hits the floor.

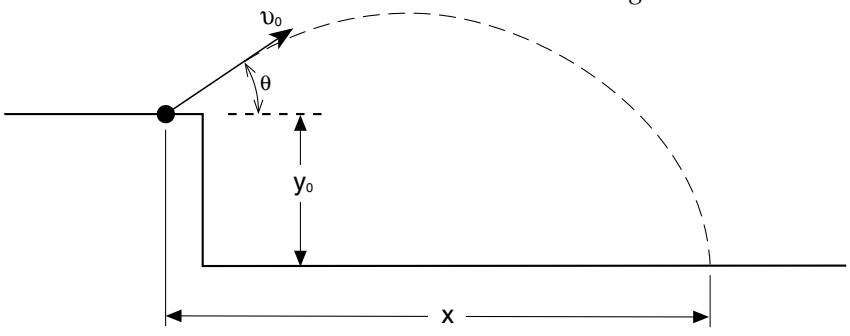


Figure 3.2 Shooting off the table

Setup

① Clamp the Projectile Launcher to a sturdy table near one end of the table with the Launcher aimed so the ball will land on the table.

② Adjust the angle of the Projectile Launcher to ten degrees.

③ Put the plastic ball into the Projectile Launcher and cock it to the medium or long range position.

➤ **NOTE:** In general, this experiment will not work as well on the short range setting because the muzzle velocity is more variable with change in angle.

④ Fire one shot to locate where the ball hits. Place a box at that location so the ball will hit at the same level as the muzzle of the launcher. See Figure 3.3.

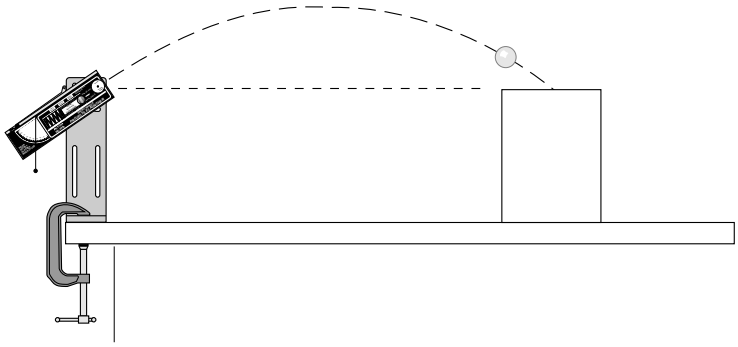


Figure 3.3 Set up to shoot on level surface

Procedure

SHOOTING ON A LEVEL SURFACE

① Fire one shot to locate where the ball hits the box. At this position, tape a piece of white paper to the box. Place a piece of carbon paper (carbon-side down) on top of this paper and tape it down. When the ball hits the box, it will leave a mark on the white paper.

② Fire about five shots.

③ Use a measuring tape to measure the horizontal distance from the muzzle to the leading edge of the paper. If a measuring tape is not available, use a plumb bob to find the point on the table that is directly beneath the release point on the barrel. Measure the horizontal distance along the table from the release point to the leading edge of the paper. Record in Table 3.1.

④ Measure from the leading edge of the paper to each of the five dots and record these distances in Table 3.1.

⑤ Increase the angle by 10 degrees and repeat all the steps.

⑥ Repeat for angles up to and including 80 degrees.

Table 3.1 Shooting on a Level Surface

Horz. Distance

Angle	10	20	30	40	50	60	70	80
1								
2								
3								
4								
5								
Average								
Paper Dist.								
Total Dist.								

SHOOTING OFF THE TABLE

Aim the projectile launcher so the ball will hit the floor. Repeat the procedure and record the data in Table 3.2.

Table 3.2 Shooting off the Table onto the Floor

Table 3.2 Shooting Off the Table

	Angle	10	20	30	40	50	60	70	80
Horz. Distance	1								
	2								
	3								
	4								
	5								
	Average								
	Paper Dist.								
	Total Dist.								

Analysis

① Find the average of the five distances in each case and record in Tables 3.1 and 3.2.

② Add the average distance to the distance to the leading edge of the paper to find the total distance (range) in each case. Record in Tables 3.1 and 3.2.

③ For each data table, plot the range vs. angle and draw a smooth curve through the points.

Questions

① From the graph, what angle gives the maximum range for each case?

② Is the angle for the maximum range greater or less for shooting off the table?

③ Is the maximum range further when the ball is shot off the table or on the level

Notes

Experiment 4: Projectile Path

EQUIPMENT NEEDED

-Projectile Launcher and plastic ball
-measuring tape or meter stick
-carbon paper
-white paper
-movable vertical target board (Must reach from floor to muzzle)
-graph paper

Purpose

The purpose of this experiment is to find how the vertical distance the ball drops is related to the horizontal distance the ball travels when the ball is launched horizontally from a table.

Theory

The range is the horizontal distance, x, between the muzzle of the Launcher and the place where the ball hits, given by $x = v_0 t$, where vo is the initial speed of the ball as it leaves the muzzle and t is the time of flight.

If the ball is shot horizontally, the time of flight of the ball will be

$$t = \frac{x}{v_0}$$

The vertical distance, y, that the ball falls in time t is given by

$$y = \frac{1}{2} g t^2$$

where g is the acceleration due to gravity.

Substituting for t into the equation for y gives

$$y = \left(\frac{g}{2{v_0}^2}\right) x^2$$

A plot of y versus x^2 will give a straight line with a slope equal to $\frac{g}{2{v_0}^2}$.

Setup

① Clamp the Projectile Launcher to a sturdy table near one end of the table with the Launcher aimed away from the table.

② Adjust the angle of the Projectile Launcher to zero degrees so the ball will be shot off horizontally.

③ Fire a test shot on medium range to determine the initial position of the vertical target. Place the target so the ball hits it near the bottom. See Figure 4.1.

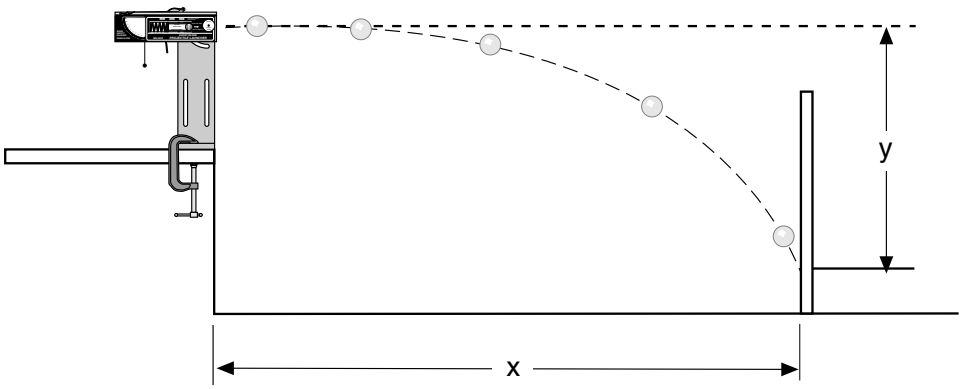


Figure 4.1 Set Up

④ Cover the target board with white paper. Tape carbon paper over the white paper.

Procedure

① Measure the vertical height from the floor to the muzzle and record in Table 4.1. Mark this height on the target.

② Measure the horizontal distance from the muzzle of the Projectile Launcher to the target and record in Table 4.1.

③ Shoot the ball.

④ Move the target about 10 to 20 cm closer to the Launcher.

⑤ Repeat Steps 2 through 4 until the height of the ball when it strikes the target is about 10 to 20 cm below the height of the muzzle.

Table 4.1 Data

Height of Muzzle = _____

Horizontal (x)	Height (y)	x^2

Analysis

① On the target, measure the vertical distances from the muzzle level mark down to the ball marks and record in Table 4.1.

② Calculate x^2 for all the data points and record in Table 4.1.

③ Plot y vs. x^2 and draw the best-fit straight line.

④ Calculate the slope of the graph and record in Table 4.2.

⑤ From the slope of the graph, calculate the initial speed of the ball as it leaves the muzzle and record in Table 4.2.

⑥ Using any data point for x and y, calculate the time using y and then calculate the initial speed using this time and x. Record the results in Table 4.2.

⑦ Calculate the percent difference between the initial speeds found using these two

Table 4.2 Initial Speed

Slope of graph	
Initial speed from slope	
Time of flight	
Initial speed from x, y	
Percent Difference	

methods. Record in Table 4.2.

Questions

① Was the line straight? What does this tell you about the relationship between y and x?

Notes

Experiment 5: Conservation of Energy

② If you plotted y vs. x, how would the graph differ from the y vs. x^2 graph?

③ What shape is the path of a projectile?

EQUIPMENT NEEDED

-Projectile Launcher and plastic ball
-measuring tape or meter stick
-(optional) 2 Photogates and Photogate Bracket
-plumb bob
-white paper
-carbon paper

Purpose

The purpose of this experiment is to show that the kinetic energy of a ball shot straight up is transformed into potential energy.

Theory

The total mechanical energy of a ball is the sum of its potential energy (PE) and its kinetic energy (KE). In the absence of friction, total energy is conserved. When a ball is shot straight up, the initial PE is defined to be zero and the KE = $(1/2)mv_0^2$, where m is the mass of the ball and vo is the muzzle speed of the ball. See Figure 5.1. When the ball reaches its maximum height, h, the final KE is zero and the PE = mgh, where g is the acceleration due to gravity. Conservation of energy gives that the initial KE is equal to the final PE.

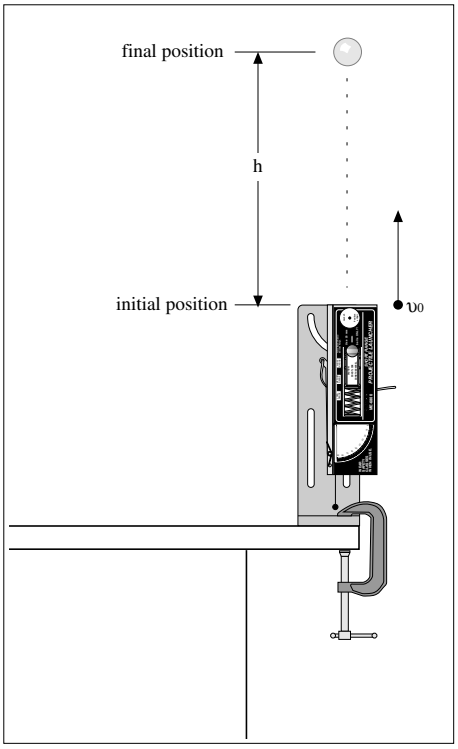


Figure 5.1 Conservation of Energy

To calculate the kinetic energy, the initial velocity must be determined. To calculate the initial velocity, vo, for a ball shot horizontally off a table, the horizontal distance travelled by the ball is given by $x = v_0 t$, where t is the time the ball is in the air. Air friction is assumed to be negligible. See Figure 5.2.

The vertical distance the ball drops in time t is given by $y = (1/2)gt^2$.

The initial velocity of the ball can be determined by measuring x and y. The time of flight of the ball can be found using

$$t = \sqrt{\frac{2y}{g}}$$

and then the initial velocity can be found using $v_0 = x/t$.

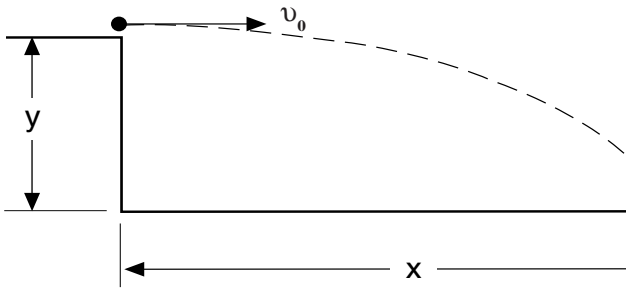


Figure 5.2 Finding the Initial Velocity

Set up

① Clamp the Projectile Launcher to a sturdy table near one end of the table with the Launcher aimed away from the table. See Figure 5.1.

② Point the Launcher straight up and fire a test shot on medium range to make sure the ball doesn't hit the ceiling. If it does, use the short range throughout this experiment or put the Launcher closer to the floor.

③ Adjust the angle of the Projectile Launcher to zero degrees so the ball will be shot off horizontally.

Procedure

PART I: Determining the Initial Velocity of the Ball (without photogates)

① Put the plastic ball into the Projectile Launcher and cock it to the medium range position. Fire one shot to locate where the ball hits the floor. At this position, tape a piece of white paper to the floor. Place a piece of carbon paper (carbon-side down) on top of this paper and tape it down. When the ball hits the floor, it will leave a mark on the white paper.

② Fire about ten shots.

③ Measure the vertical distance from the bottom of the ball as it leaves the barrel (this position is marked on the side of the barrel) to the floor. Record this distance in Table 5.1.

④ Use a plumb bob to find the point on the floor that is directly beneath the release point on the barrel. Measure the horizontal distance along the floor from the release point to the leading edge of the paper. Record in Table 5.1.

⑤ Measure from the leading edge of the paper to each of the ten dots and record these distances in Table 5.1.

⑥ Find the average of the ten distances and record in Table 5.1.

⑦ Using the vertical distance and the average horizontal distance, calculate the time of flight and the initial velocity of the ball. Record in Table 5.1.

Trial Number	Distance
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Average	
Total Distance	

Table 5.1 Determining the Initial Velocity without Photogates

Vertical distance = _____ Calculated time of flight= _____

Horizontal distance to paper edge = _____ Initial velocity = _____

ALTERNATE METHOD FOR DETERMINING THE INITIAL VELOCITY OF THE BALL (USING PHOTOGATES)

① Attach the photogate bracket to the Launcher and attach two photogates to the bracket. Plug the photogates into a computer or other timer.

② Adjust the angle of the Projectile Launcher to 90 degrees (straight up).

③ Put the plastic ball into the Projectile Launcher and cock it to the long range position.

④ Run the timing program and set it to measure the time between the ball blocking the two photogates.

TRIAL NUMBER	TIME
1	
2	
3	
AVERAGE TIME	
INITIAL SPEED	

⑤ Shoot the ball three times and take the average of these times. Record in Table 5.2.

Table 5.2 Initial Speed Using Photogates

⑥ Using that the distance between the photogates is 10 cm, calculate the initial speed and record it in Table 5.2.

MEASURING THE HEIGHT

① Adjust the angle of the Launcher to 90 degrees (straight up).

② Shoot the ball on the medium range setting several times and measure the maximum height attained by the ball. Record in Table 5.3.

③ Determine the mass of the ball and record in Table 5.3.

Analysis

Table 5.3 Results

Maximuim Height of Ball	
Mass of Ball	
Initial Kinetic Energy	
Final Potential Energy	
Percent Difference	

① Calculate the initial kinetic energy and record in Table 5.3.

② Calculate the final potential energy and record in Table 5.3.

③ Calculate the percent difference between the initial and final energies and record in Table 5.3.

Questions

Experiment 6: Conservation of Momentum In Two Dimensions

EQUIPMENT NEEDED

-Projectile Launcher and 2 plastic balls
-meter stick
-butcher paper
-stand to hold ball
-plumb bob
-protractor
-tape to make collision inelastic
-carbon paper

Purpose

The purpose of this experiment is to show that the momentum is conserved in two dimensions for elastic and inelastic collisions.

Theory

A ball is shot toward another ball which is initially at rest, resulting in a collision after which the two balls go off in different directions. Both balls are falling under the influence of the force of gravity so momentum is not conserved in the vertical direction. However, there is no net force on the balls in the horizontal plane so momentum is conserved in horizontal plane.

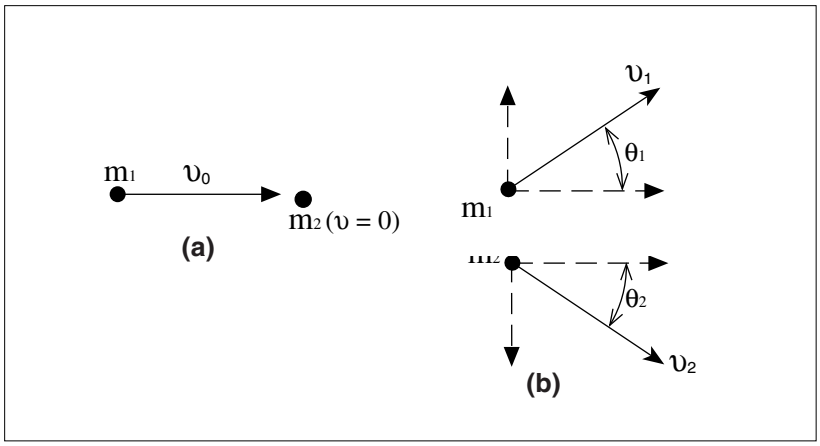


Figure 6.1: (a) Before Collision (b) After Collision

Before the collision, since all the momentum is in the direction of the velocity of Ball #1 it is convenient to define the x-axis along this direction. Then the momentum before the collision is

$$\vec{P}_{before} = m_1 v_0 \,\hat{x}$$

and the momentum after the collision is

$$\vec{P}_{after} = \left(m_1 v_{1x} + m_2 v_{2x}\right)\hat{x} + \left(m_1 v_{1y} - m_2 v_{2y}\right)\hat{y}$$

where $v_{1x} = v_1 \cos\theta_1$, $v_{1y} = v_1 \sin\theta_1$, $v_{2x} = v_2 \cos\theta_2$ and $v_{2y} = v_2 \sin\theta_2$

Since there is no net momentum in the y-direction before the collision, conservation of momentum requires that there is no momentum in the y-direction after the collision. Therefore,

$$m_1 v_{1y} = m_2 v_{2y}$$

Equating the momentum in the x-direction before the collision to the momentum in the x-direction after the collision gives

$$m_1 v_0 = m_1 v_{1x} + m_2 v_{2x}$$

In an elastic collision, energy is conserved as well as momentum.

$$\frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

① How does friction affect the result for the kinetic energy?

② How does friction affect the result for the potential energy?

Also, when energy is conserved, the paths of two balls (of equal mass) after the collision will be at right angles to each other.

Set up

① Clamp the Projectile Launcher to a sturdy table near one end of the table with the Launcher aimed inward toward the table.

② Adjust the angle of the Projectile Launcher to zero degrees so the ball will be shot off horizontally onto the table. Fire a test shot on the short range setting to make sure the ball lands on the table.

③ Cover the table with butcher paper. The paper must extend to the base of the Launcher.

④ Mount collision attachment on the Launcher. See Figure 6.2. Slide the attachment back along the Launcher until the tee is about 3 cm in front of the muzzle.

⑤ Rotate the attachment to position the ball from side to side. The tee must be located so that neither ball rebounds into the Launcher and so both balls land on the table. Tighten the screw to secure the collision attachment to the Launcher.

⑥ Place a piece of carbon paper at each of the three sites where the balls will land.

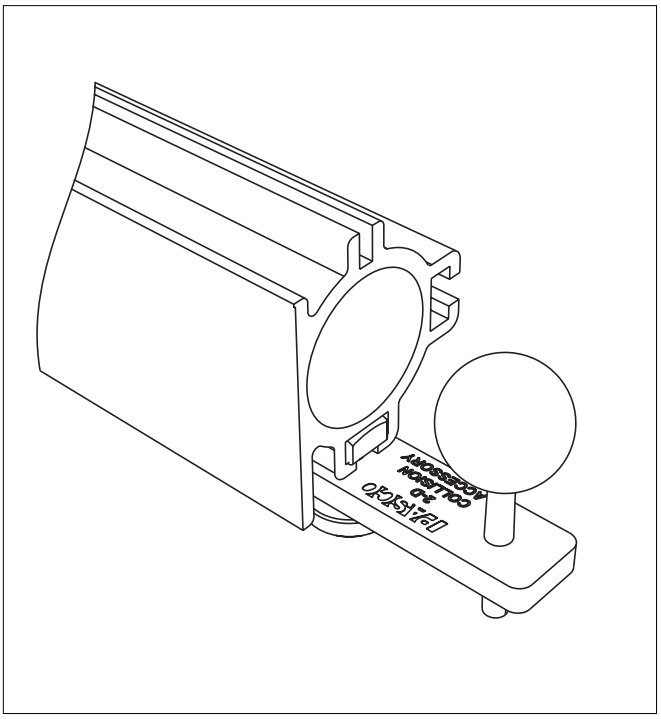


Figure 6.2: Photogate Bracket and Tee

Procedure

① Using one ball, shoot the ball straight five times.

② Elastic collision: Using two balls, load one ball and put the other ball on the tee. Shoot the ball five times.

③ Inelastic collision: Using two balls, load one ball and stick a very small loop of tape onto the tee ball. Orient the tape side of the tee ball so it will be struck by the launched ball, causing an inelastic collision. Shoot the ball once and if the balls miss the carbon paper, relocate the carbon paper and shoot once more. Since the tape does not produce the same inelastic collision each time, it is only useful to record this collision once.

④ Use a plumb bob to locate on the paper the spot below the point of contact of the two balls. Mark this spot.

Analysis

① Draw lines from the point-of-contact spot to the centers of the groups of dots. There will be five lines.

② Measure the lengths of all five lines and record on the paper. Since the time of flight is the same for all paths, these lengths are proportional to the corresponding horizontal velocities. Since the masses are also the same, these lengths are also proportional to the corresponding momentum of each ball.

③ Measure the angles from the center line to each of the outer four lines and record on the paper.

PERFORM THE FOLLOWING THREE STEPS FOR THE ELASTIC COLLISION AND THEN REPEAT THESE THREE STEPS FOR THE INELASTIC COLLISION:

④ For the x-direction, check that the momentum before equals the momentum after the collision. To do this, use the lengths for the momentums and calculate the x-components using the angles. Record the results in Tables 6.1 and 6.2.

Table 6.1 Results for the Elastic Collision

Initial x-momentum		Final x-momentum		% difference	
y-momentum ball 1		y-momentum ball 2		% difference	
Initial KE		Final KE		% difference	

Table 6.2 Results for the Inelastic Collision

Initial x-momentum		Final x-momentum		% difference	
y-momentum ball 1		y-momentum ball 2		% difference	
Initial KE		Final KE		% difference	

⑤ For the y-direction, check that the momenta for the two balls are equal and opposite, thus canceling each other. To do this, calculate the y-components using the angles. Record the results in the Tables.

⑥ Calculate the total kinetic energy before and the total kinetic energy after the collision. Calculate the percent difference. Record the results in the Tables.

Questions

① Was momentum conserved in the x-direction for each type of collision?

② Was momentum conserved in the y-direction for each type of collision?

③ Was energy conserved for the elastic collision?

③ Test fire the ball (on the long range setting) a few times to find approximately what angle gives the maximum height on the wall. (NOTE: In general, this experiment will not work as well on the short range setting because the muzzle velocity is more variable with change in angle.)

④ Tape a piece of white paper to the board in the region where the ball is hitting. Then

Experiment 7: Varying Angle To Maximize Height on a Wall

EQUIPMENT NEEDED

-Projectile Launcher and plastic ball
-measuring tape or meter stick
-white paper
-plumb bob
-carbon paper
-board to protect wall

Purpose

The purpose of this experiment is to find the launch angle which will maximize the height on a vertical wall for a ball launched at a fixed horizontal distance from the wall.

Theory

When the ball is shot at an angle at a fixed distance, x, from a vertical wall, it hits the wall at a height y given by:

$$y = y_0 + (v_0 \sin\theta)\, t - \frac{1}{2} gt^2$$

where y_0 is the initial height of the ball, v_0 is the initial speed of the ball as it leaves the muzzle, θ is the angle of inclination above horizontal, g is the acceleration due to gravity, and t is the time of flight. The range is the horizontal distance, x, between the muzzle of the Launcher and the place where the ball hits, given by $x = (v_0 \cos\theta)t$. Solving for the time of flight from the equation for x gives

$$t = \frac{x}{v_0 \cos\theta}$$

Substituting for t in the equation for y gives

$$y = y_0 + x \tan\theta - \frac{gx^2}{2{v_0}^2 \cos^2\theta}$$

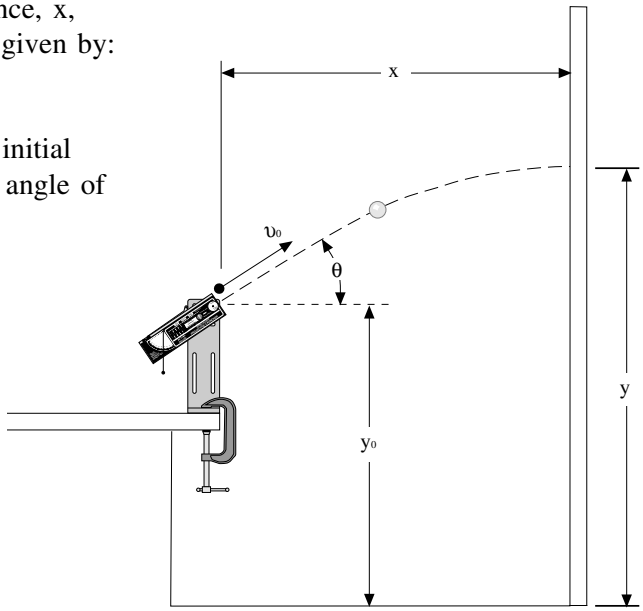


Figure 7.1 Maximizing Height

To find the angle that gives the maximum height, y, set dy/dθ equal to zero and solve for the angle.

$$\frac{dy}{d\theta} = x \sec^2\theta - \frac{gx^2 \tan\theta \sec^2\theta}{{v_0}^2} = 0$$

Solving for the angle gives

$$\tan\theta_{max} = \frac{{v_0}^2}{gx}$$

Since the second derivative is negative for θ_{max}, the angle is a maximum.

To find the initial velocity of the ball, the fixed distance x and the maximum height y_{max} can be used. Solve the y-equation for v_0 and plug in the values for y_{max}, θ_{max}, and x.

Set up

① Clamp the Projectile Launcher to a sturdy table near one end of the table with the Launcher facing the wall at a distance of about 2 meters from the wall.

② Put a vertical board up to protect the wall.

cover the white paper with a piece of carbon paper.

Procedure

① Shoot the ball at various angles and pinpoint exactly which angle gives the maximum height by checking the marks on the paper.

② Measure the angle that produces the maximum height and record in Table 7.1.

③ Measure the maximum height and record in Table 7.1.

④ Measure the horizontal distance from the muzzle to the vertical board and record in Table 7.1.

⑤ Measure the initial height of the ball where it leaves the muzzle and record in Table 7.1.

Analysis

① Calculate the initial velocity by solving the y-equation for v_0 and plugging in the values from Table 7.1.

Table 7.1 Data and Results

Measured Angle for Max	
Maximum Height	
Horizontal Distance	
Initial Height	
Calculated Initial Velocity	
Calculated Angle for Max	
% Difference Between Angles	

② Calculate the angle for maximum height using the initial velocity calculated in Step 1 and the horizontal distance from the wall to the launcher.

③ Calculate the percent difference between the measured angle and the calculated angle.

Questions

① For the angle which gives the maximum height, when the ball hits the wall, has it already reached the peak of its trajectory?

② For what distance from the wall would the height be maximized at 45·? What would the maximum height be in this case?

EQUIPMENT NEEDED

-Projectile Launcher and steel ball

-box to make elevation same as muzzle

Purpose

Experiment 8 (Demo): Do 30° and 60° Give the Same Range?

The purpose of this demonstration is to show that the range of a ball launched at 30· is the same as one launched at 60· if the ball is shot on a level surface.

Theory

The range is the horizontal distance, x, between the muzzle of the Launcher and the place where the ball hits, given by $x = (v_0 \cos\theta)\, t$ where v_0 is the initial speed of the ball as it leaves the muzzle, θ is the angle of inclination above horizontal, and t is the time of flight.

If the ball hits on a place that is at the same level as the level of the muzzle of the launcher, the time of flight of the ball will be twice the time it takes the ball the reach the peak of its trajectory:

$$t = 2t_{peak} = 2\,\frac{v_0 \sin\theta}{g}$$

where g is the acceleration due to gravity.

Substituting for t into the equation for x gives

$$x = \frac{2{v_0}^2 \sin\theta \cos\theta}{g}$$

and using a trigonometry identity gives

$$x = \frac{{v_0}^2 \sin 2\theta}{g}$$

The ranges for the angles 30° and 60° are the same since sin(60°) = sin(120°).

Set up

① Clamp the Projectile Launcher to a sturdy table near one end of the table with the Launcher aimed so the ball will land on the table.

② Adjust the angle of the Projectile Launcher to 30 degrees.

③ Put the steel ball into the Projectile Launcher and cock it to the medium or long range position.

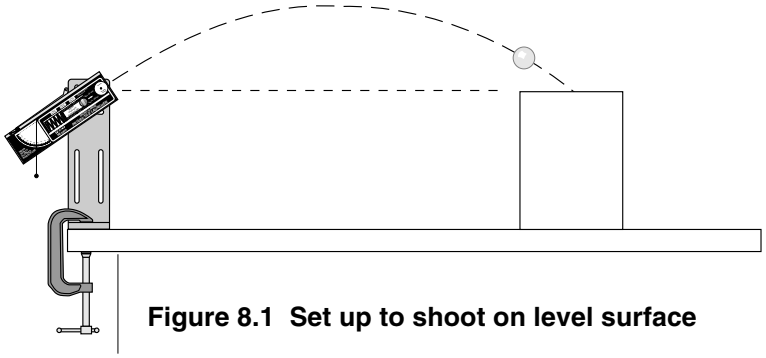


Figure 8.1 Set up to shoot on level surface

➤ **NOTE:** In general, this experiment will not work as well on the short range setting because the muzzle velocity is more variable with change in angle.

Fire one shot to locate where the ball hits. Place an inverted box at that location so the ball will hit at the same level as the muzzle of the launcher. See Figure 10.1.

Procedure

① Shoot the ball at 30 degrees to demonstrate that the ball lands on the box.

② Change the angle of the Launcher to 60 degrees and shoot the ball again. Call attention to the fact that the ball again lands on the box. Thus the ranges are the same.

③ Change the angle to 45 degrees and shoot the ball again to show that the ball now lands further away, missing the box.

④ Ask the question: What other pairs of angles will have a common range? This demonstration can be done for any two angles which add up to 90 degrees: 20 and 70, or 35 and 55, etc.

EQUIPMENT NEEDED

-2 Projectile Launchers and 2 plastic balls

Purpose

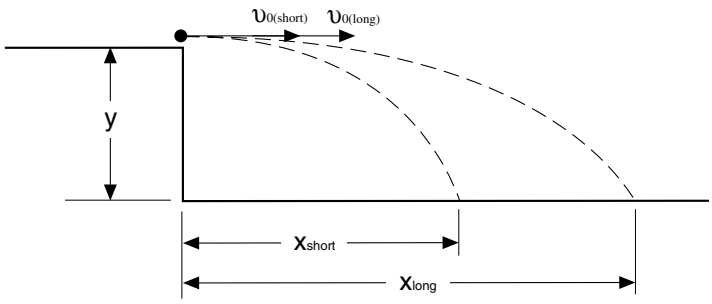
Experiment 9 (Demo): Simultaneously Shoot Two Balls Horizontally at Different Speeds

The purpose of this demonstration is to show that regardless of the initial speed of the balls launched horizontally off a table, the balls will hit the floor at the same time.

Theory

Two balls are shot off horizontally from the same table (from the same height, y). The muzzle speeds of the two balls are different.

The vertical and horizontal motions of a projectile are independent of each other. The horizontal distance, x, travelled by the ball is dependent on the initial speed, v_0, and is given by $x = v_0 t$, where t is the time of flight. The time of flight depends only on the vertical distance the ball falls since $y = (1/2)gt^2$. Since the vertical distance is the same each ball, the time of flight must be the same for each ball.



Set up

① Clamp two Projectile Launchers adjacent to each other on a sturdy table. The Launchers should both be aimed in the same direction, away from the table so the balls will land on the floor.

② Adjust the angle of each Projectile Launcher to zero degrees so the balls will be shot horizontally off the table.

Procedure

① Put a plastic ball into each Projectile Launcher and cock one Launcher to the short range position and cock the other Launcher to the long range position.

② Ask the class to be quiet and listen for the balls striking the floor. Tell them if they hear only one click, that means the balls hit the floor simultaneously.

③ Put both lanyards in the same hand and pull them at the same time so the balls are launched simultaneously.

④ After the balls hit the floor, ask the class if they heard one click or two.

EQUIPMENT NEEDED

-Projectile Launcher and plastic ball
-2 Photogates
-Meter stick
-5 ring clamps on stands
-Photogate Bracket
- 2 Meter stick

Purpose

The purpose of this demonstration is to show that the path of a ball launched horizontally from a table is parabolic.

Notes

Experiment 10 (Demo):ShootingThrough Hoops

Theory

The range is the horizontal distance, x, between the muzzle of the Launcher and the place where the ball hits, given by

$$x = v_0 t$$

where vo is the initial speed of the ball as it leaves the muzzle and t is the time of flight.

The vertical position, y, of the ball at time t is given by

$$y = \frac{1}{2} gt^2$$

where y_0 is the initial height of the ball and g is the acceleration due to gravity.

Set up

① Before the demonstration begins, find the initial velocity for the range setting to be used. Attach the photogates and use a computer to find the initial velocity or shoot the ball horizontally and measure x and y to find the initial velocity. See experiments 1 and 2.

② To prepare to demonstrate, clamp the Projectile Launcher to the demonstration table with the Launcher aimed away from the table so the ball will land on the floor.

③ Adjust the angle of the Launcher to zero degrees so it will shoot horizontally.

Procedure

① In front of the class, measure the initial height of the ball at muzzle level.

② Calculate the horizontal and vertical positions of the ball each 1/10 second until it hits the floor.

③ Lay the 2-meter stick on the floor in a straight line away from the Launcher. Remove the back mounting screw from the Launcher base so the back of the Launcher can be rotated upward. Look through the back of the Launcher and align the sights and the end of the 2m stick so the 2m stick is aligned with the path of the ball. Relevel the Launcher.

④ Measure off each set of x and y and place a ring clamp on a stand at each position (See Figure 10.1). If possible it is best to adjust the last two ring stands at an angle from the vertical so the ball will not have to pass through them at an oblique angle. A cup may be placed at the end of

t (sec)	$x = v_0 t$ (cm)	$y = y_0 - (1/2)gt^2$ (cm)
0.1		
0.2		
0.3		
0.4		
0.5		

the path to catch the ball.

⑤ Shoot the ball through the rings.

⑥ Ask the class what shape of curve is formed by the rings.

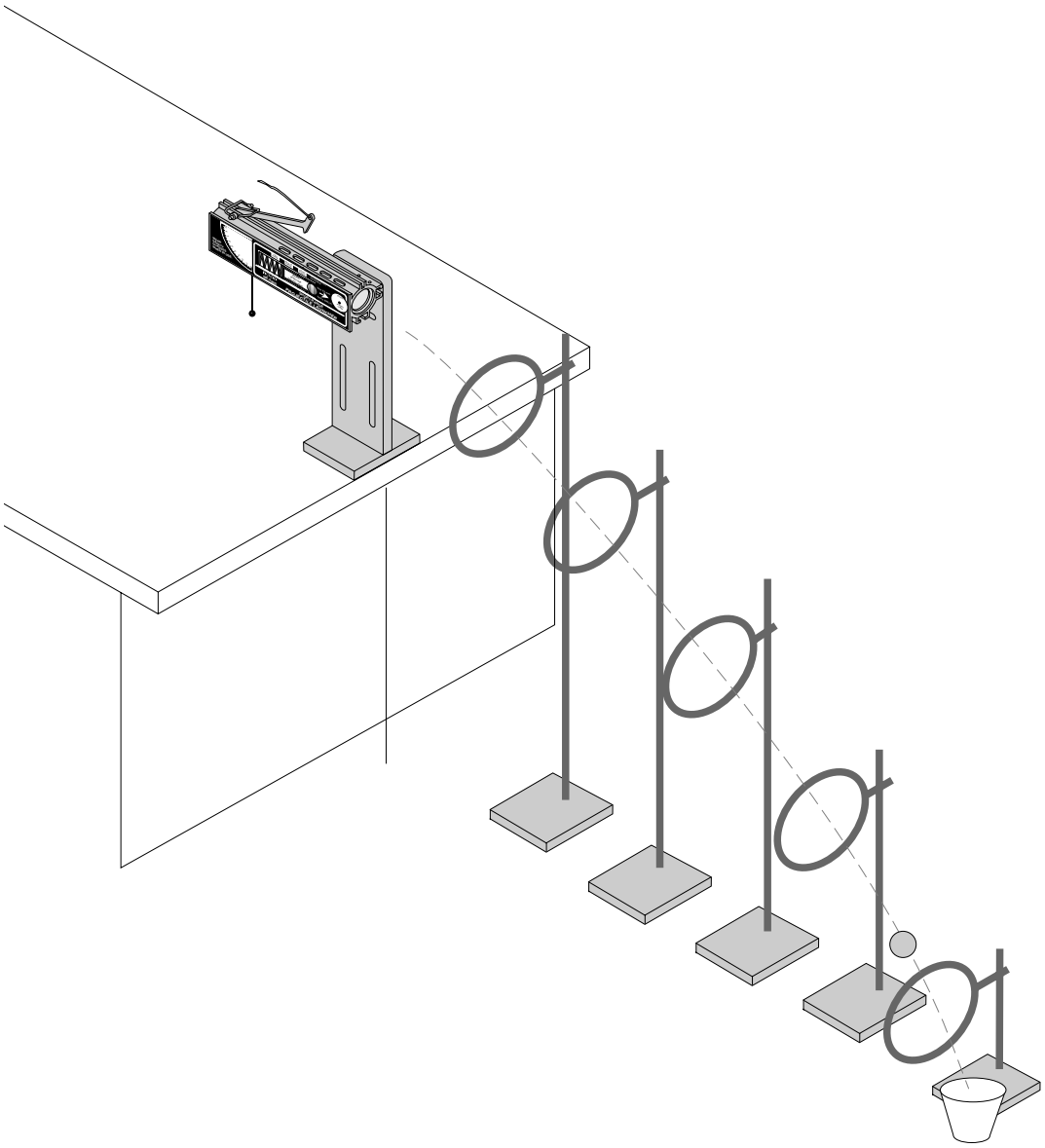


Figure 10.1 Placing the rings

Teacher's Guide

Experiment 1: Projectile Motion

Procedure

➤ **NOTE:** For best results, make sure that the projectile launcher is clamped securely to a firm table. Any movement of the gun will result in inconsistent data.

A) The muzzle velocity of the gun tested for this manual was 6.5 m/s (Short range launcher at maximum setting, nylon ball)

B) To find the range at the chosen angle, it is necessary to solve the quadratic equation given in the theory section. You may wish for the students to do this, or you may provide them with the solution:

$$t=\frac{v_0\sin\theta+\sqrt{(v_0\sin\theta)^2+2g(y_0-y)}}{g}$$

Analysis

① The difference depended on the angle at which the gun was fired. The following table gives typical results:

Angle	Predicted Range	Actual Range	Percent Error
30	5.22	5.19	0.57%
45	5.30	5.16	2.64%
60	4.35	4.23	2.87%
39	5.39	5.31	1.48%

➤ **NOTE:** The maximum angle is not 45° in this case, nor is the range at 60° equal to that at 30°. This is because the initial height of the ball is not the same as that of the impact point. The maximum range for this setup (with the launcher 1.15 m above ground level) was calculated to be 39°, and this was experimentally verified as well.

② Answers will vary depending on the method of estimating the precision. The primary source of error is in ignoring the effect of air resistance.

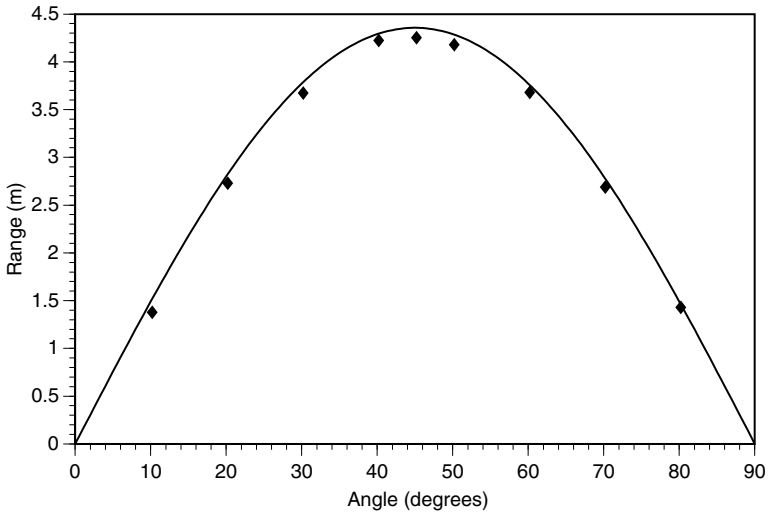
Experiment 2: Projectile Motion Using Photogates

➤ **NOTE:** Other than the method of determining initial velocity, this experiment and experiment 1 are equivalent.

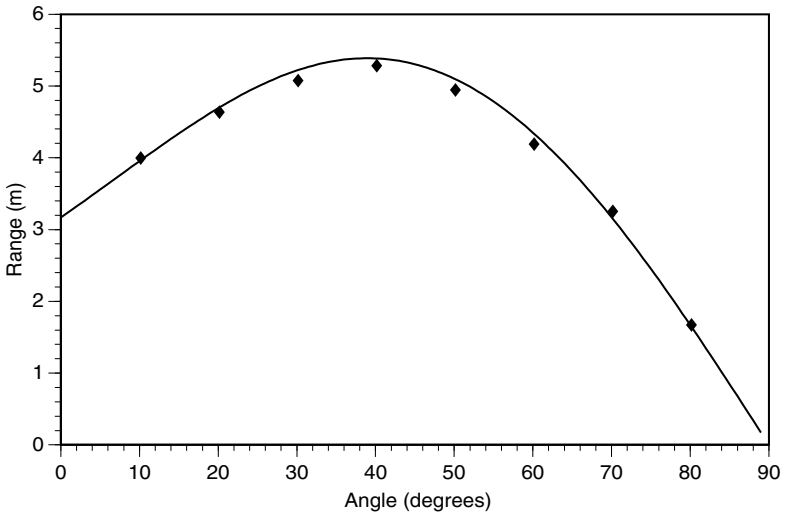
Experiment 3: Projectile Range Versus Angle

Procedure

Shooting off a level surface:



Shooting off a table:



➤ **NOTE:** The curves shown are for the calculated ranges in each case. The data points are the actual measured ranges.

Questions:

① On a level surface, the maximum range is at 45°. For a non-level surface, the angle of maximum range depends on the initial height of the projectile. For our experimental setup, with an initial height of 1.15 m, the maximum range is at 40°. (Theoretical value 39°)

② The angle of maximum range decreases with table height.

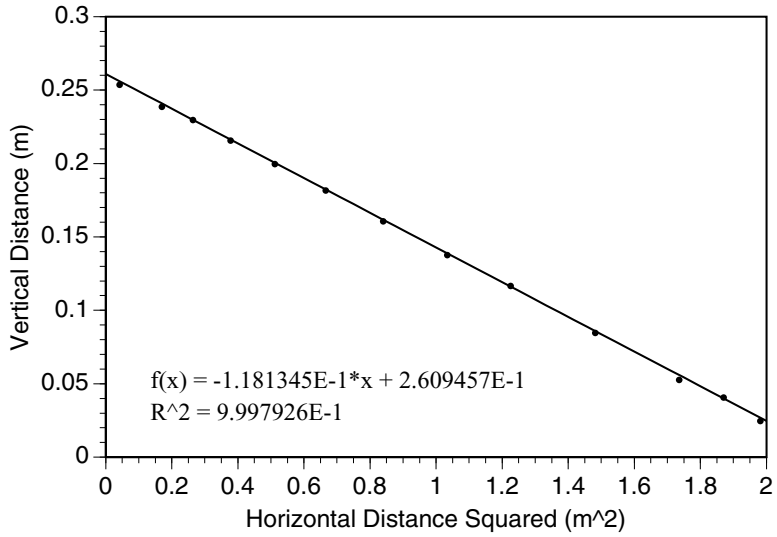
③ The maximum distance increases with table height.

Experiment 4: Projectile Path

Analysis

① Alternately, measure your distances from the ground up.

② Vertical distances measured from the ground up for this graph. The intercept is the height of the launcher above ground when done this way.



③ The slope (measuring from the ground) is -0.118 for this test. (Measuring down from the initial height will give the same value, only positive.) In either case, the slope is

$$\frac{g}{2v_0^2}$$

④ The slope calculated here gives us an initial velocity of 6.44 m/s. This compares favorably with the velocity calculated in experiments 1 and 2.

Questions

① Yes. This tells us that y is a function of x^2.

② A plot of y versus x would be parabolic instead of linear.

③ The projectile moves in a parabolic curve. (neglecting air friction)

Experiment 5: Conservation of Energy

Analysis

① Using the photogate method, we found that the initial speed of the ball was 4.93 m/s. (Nylon ball, short range launcher at medium setting) The ball mass was 9.6 g, so our total kinetic energy was 0.117 J.

② The ball reached an average height of 1.14 m. Potential energy was then 0.107 J.

③ Energy lost was 8.5% of original energy.

Experiment 6: Conservation of Momentum in Two Dimensions

Setup

① If possible use medium range rather than short. The medium-range setting gives more predictable results than the short-range setting.

Analysis

① Results for the x component of momentum should be within 5% of initial values. The total y component should be small compared to the x component.

Questions

① Momentum is conserved on both axes.

② Kinetic energy is nearly conserved in the elastic collision. There is some loss due the fact that the collision is not completely elastic.

③ Energy is conserved for the inelastic collision; but kinetic energy is not.

④ The angle should be nearly 90°. (Our tests had angles of about 84°)

⑤ In the inelastic case, the angle will be less than in the elastic case. The exact angle will depend on the degree of inelasticity, which will depend on the type and amount of tape used.

Experiment 7: Varying Angle to Maximize Height on a Wall

Procedure

① You should be able to measure the angle of maximum height to within ±2%.

② Measure the distance to the front edge of the ball.

③ Measure the initial height to the center of the ball.

Analysis

① The initial velocity should be close to the initial velocity determined by other methods. You may wish to determine the initial velocity by the method in lab 1, and use that value in your calculations for the rest of this experiment.

② Measured and calculated should agree to within 3%.

Questions

① The ball will have passed its peak by the time it reaches the wall. To show this, take the derivative of y with respect to x:

$$y = y_0 + x\tan\theta_{max} - \frac{gx^2}{2v_0^2\cos^2\theta_{max}}$$

$$\frac{dy}{dx} = \tan\theta_{max} - \frac{gx}{v_0^2\cos^2\theta_{max}}$$

substitute $\theta_{max} = \tan^{-1}\left(\frac{v_0^2}{gx_{max}}\right)$

$$\frac{dy}{dx} = \frac{v_0^2}{gx_{max}} - \frac{gx}{v_0^2\cos^2\left[\tan^{-1}\left(\frac{v_0^2}{gx_{max}}\right)\right]}$$

Substitute $\cos\left[\tan^{-1}\left(\frac{a}{b}\right)\right] = \frac{b}{\sqrt{a^2+b^2}}$ and simplify.

$$\frac{dy}{dx} = \frac{v_0^2}{gx_{max}} - \frac{gx}{v_0^2\left(\frac{gx_{max}}{\sqrt{v_0^4 + g^2x_{max}^2}}\right)^2} = \frac{v_0^2}{gx_{max}} - \frac{x\left(v_0^4 + g^2x_{max}^2\right)}{v_0^2gx_{max}^2}$$

$$\frac{dy}{dx} = \frac{v_0}{gx_{max}} - \frac{v_0\ x}{gx_{max}^2} - \frac{xg}{v_0^2}$$

When x = x_{max}, the value of this derivative is negative.

$$\left.\frac{dy}{dx}\right|_{x_{max}} = -\frac{gx_{max}}{v_0^2}$$

Therefore, the ball has already reached the peak and is on its way down.

② Solve the equation for maximum angle to determine x.

$$\tan\theta_{max} = \frac{v_0^2}{gx} \Rightarrow x = \frac{v_0^2}{g}$$

Substitute this value into the equation for y to determine the maximum height.

$$y = y_0 + \frac{v_0^2}{g} - \frac{g\left(\frac{v_0^2}{g}\right)^2}{v_0^2} = y_0 + \frac{v_0^2}{g} - \frac{v_0^2}{g}$$

$$y = y_0$$

Notes